The Design of Digital PID Controllers using Emulation and Direct Techniques

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Electrical Engineering 360
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April 28, 2000

Introduction

In electrical engineering, compensators are often used to stabilize or otherwise improve the response of a control system. The most common form of compensator is the PID controller. Mathematically, a PID controller consists of three terms: proportional, integrator, and derivative. The PID is a form of the lag-lead compensator design. This paper explores the development of a digital PID controller design for a given system using two different techniques: emulation and direct.

Plant Transfer Function

The first step in the design process was to develop an open loop mathematical model based on the given description of the system. The system consisted of a DC servomotor positioning an antenna platform through 30:1 reduction gears. The system also included a power amplification of 10. A general transfer function for a servomotor was located in Section 2.8 of *Feedback Control Systems* by Phillips and Harbor. This transfer function was:

$$G(s) = \frac{K_t}{JL_m s^3 + (BL_m + JR_m)s^2 + (BR_m + K_t K_m)s}$$

All values needed to complete the transfer function were given except for J; however, J was calculated by using the given values J_m and J_l. The value for J was calculated using the following equation:

$$J = J_m + (\tfrac{1}{30})^2 J_l$$

This equation was used because the J_m was the moment of inertia of servomotor rotor and J_l was the moment of inertia of the antenna platform. Since the antenna platform was on the opposite side of the 30:1 reduction gears, its moment of inertia was decreased. The value of J shows the moment of inertia for the entire mechanical portion of the system. After including the power amplification of 10 and the effects of the 30:1 reduction gears the open loop transfer function of the plant was determined to be:

$$G_p(s) = \frac{13.33}{4.133s^3 + 83.66s^2 + 36s}$$

The model that was developed was used in both design techniques.

Emulation Method

The first design technique to be used was the emulation design technique. Using this method, a model for the compensator is first developed in the s-domain and then transformed into the z-domain for implementation in a digital computer. The sampling frequency for the system needs to be 30 times the bandwidth of the compensated system in order to receive excellent results.

The first step in the design of the compensator is to determine $?_1$. $?_1$ is then used to provide information required to compute the gain of each term of the PID compensator. For this system, $?_1$ must have a phase angle that is less than -125°. This is because the compensated system is required to have a phase-margin of 50°. The response of the system was determined using Matlab (see attached calculations) in order to find a suitable *? 1*. The first value of *?* that can satisfy the condition is 0.3072; however, the larger value of 1.1758 was chosen in hopes that it would better meet the steady-state requirements of the system. Using equations found in *Feedback Control Systems*, values for ?, K_p, and K_d were calculated. K_i was chosen to be 0.001. The continuous time transfer function for the compensator was determined to be:

$$G_c(s) = \frac{4.191s^2 + 7.62s + 0.001}{s}$$

The frequency and step response was then examined (see attached sheets). The system appeared to meet the system design criteria. The compensator was then transformed to the z-domain using the Tustin method:

$$G_c(z) = \frac{54.68z^2 - 94.12z + 39.44}{z^2 - 1}$$

A sampling period of 0.1781 seconds was used. This value was determined by examining the bode plot of the system in order to find the bandwidth.

Direct Method

Next a direct method for determining the transfer function of the compensator was used. With the direct method, the plant transfer function must be converted into the z-domain. This conversion was done using the zero-order-hold method with a sampling period 0.3 seconds in Matlab. 0.3 seconds was used to meet the system design requirements. The system was then converted into the continuous time w-domain using the Tustin method in Matlab. The design of the PID controller was based on the characteristics of the w-domain transfer function.

A method similar to that described above was used to determine the gain for each term of the w-domain compensator (see attached Matlab calculations). *? 1* was again chosen to be 1.1758. The continuous time transfer function for the compensator was found to be:

$$G_c(w) = \frac{5.161w^2 + 6.582w + 0.001}{w}$$

The Tustin method was used with a sampling period of 0.3 seconds to convert the system into a discrete time z-domain system. The frequency and step response of the system was plotted and found to be acceptable. The z-domain transfer function of the controller was found to be:

$$G_c(z) = \frac{40.99z^2 - 68.82z + 27.83}{z^2 - 1}$$

Results

This project was designed to compare the results between two different design techniques for digital controllers. The results of the two design procedures were then compared using Matlab and Simulink. Both the Matlab and Simulink tests showed that the results of the two design methods were nearly identical.

The attached sheets contain Matlab calculation and figures that were used to design the two PID compensators.